The following difference equation is given: an=4an−1−3an−2a\_n=4a\_{n-1}-3a\_{n-2}an​=4an−1​−3an−2​. What is the auxiliary or the characteristic equation?



**λ2−4λ+3=0\lambda^2-4\lambda+3=0λ2−4λ+3=0**



λ2+4λ−3=0\lambda^2+4\lambda-3=0λ2+4λ−3=0



an−4an−1+3an−2=0a\_n-4a\_{n-1}+3a\_{n-2}=0an​−4an−1​+3an−2​=0

Question 2

1

Point

**2. Question 2**

Solve the difference equation an=4an−1−3an−2a\_n=4a\_{n-1}-3a\_{n-2}an​=4an−1​−3an−2​.



**an=c1+c23na\_n=c\_1+c\_2 3^nan​=c1​+c2​3n**

****

**~~an=1+3na\_n=1+3^nan​=1+3n~~**

****

**an=c13n+c2a\_n=c\_1 3^n+c\_2an​=c1​3n+c2​**

Question 3

1

Point

**3. Question 3**

Solve the difference equation an=4an−1−3an−2a\_n=4a\_{n-1}-3a\_{n-2}an​=4an−1​−3an−2​ with initial data a0=2a\_0=2a0​=2 and a1=−2a\_1=-2a1​=−2.



**an=4−2(3n)a\_n=4- 2 (3^n)an​=4−2(3n)**



an=6−4(2n)a\_n=6- 4 (2^n)an​=6−4(2n)

Question 4

1

Point

**4. Question 4**

Stationary AR(1) process is given: Xt=0.4Xt−1+ZtX\_t=0.4X\_{t-1}+Z\_tXt​=0.4Xt−1​+Zt​. Find the Yule-Walker equations.



**~~ρ(k)=0.4ρ(k−1)\rho(k)=0.4\rho(k-1)ρ(k)=0.4ρ(k−1) for all k∈Zk\in \mathbb{Z}k∈Z.~~**



(1−0.4B)Xt=Zt(1-0.4B)X\_t=Z\_t(1−0.4B)Xt​=Zt​



**ρ(k)=0.4ρ(k−1)\rho(k)=0.4\rho(k-1)ρ(k)=0.4ρ(k−1) when k≥1k\geq1k≥1.**

Question 5

1

Point

**5. Question 5**

Find the solution of the Yule-Walker equations of the process Xt=0.4Xt−1+ZtX\_t=0.4X\_{t-1}+Z\_tXt​=0.4Xt−1​+Zt​.



**~~ρ(k)=c0.4k\rho(k)=c 0.4^kρ(k)=c0.4k for k≥1k\geq 1k≥1.~~**



**ρ(k)=0.4k\rho(k)= 0.4^kρ(k)=0.4k for k≥0k\geq 0k≥0.**



**ρ(k)=0.4k\rho(k)= 0.4^kρ(k)=0.4k for k≥0k\geq 0k≥0, and ρ(k)=ρ(−k)\rho(k)=\rho(-k)ρ(k)=ρ(−k) for k∈Z−k\in \mathbb{Z}^-k∈Z−.**

Question 6

1

Point

**6. Question 6**

Find the Yule-Walker equations and general solutions of them that govern autocorrelation coefficients of the AR(3) process

Xt=12Xt−1+19Xt−2−118Xt−3+ZtX\_t=\frac{1}{2}X\_{t-1}+\frac{1}{9}X\_{t-2}-\frac{1}{18}X\_{t-3}+Z\_tXt​=21​Xt−1​+91​Xt−2​−181​Xt−3​+Zt​



ρ(k)=12ρ(k−1)+19ρ(k−2)−118ρ(k−3)\rho(k)=\frac{1}{2}\rho(k-1)+\frac{1}{9}\rho(k-2)-\frac{1}{18}\rho(k-3)ρ(k)=21​ρ(k−1)+91​ρ(k−2)−181​ρ(k−3)

ρ(k)=c1(12)k+c2(19)k+c3(−118)k\rho(k)=c\_1 (\frac{1}{2})^k+c\_2 (\frac{1}{9})^k+c\_3 (-\frac{1}{18})^kρ(k)=c1​(21​)k+c2​(91​)k+c3​(−181​)k



**ρ(k)=12ρ(k−1)+19ρ(k−2)−118ρ(k−3)\rho(k)=\frac{1}{2}\rho(k-1)+\frac{1}{9}\rho(k-2)-\frac{1}{18}\rho(k-3)ρ(k)=21​ρ(k−1)+91​ρ(k−2)−181​ρ(k−3)**

**ρ(k)=c1(12)k+c2(13)k+c3(−13)k\rho(k)=c\_1 (\frac{1}{2})^k+c\_2 (\frac{1}{3})^k+c\_3 (-\frac{1}{3})^kρ(k)=c1​(21​)k+c2​(31​)k+c3​(−31​)k**



18ρ(k)=9ρ(k−1)+2ρ(k−2)−ρ(k−3)18\rho(k)=9\rho(k-1)+2\rho(k-2)-\rho(k-3)18ρ(k)=9ρ(k−1)+2ρ(k−2)−ρ(k−3)

ρ(k)=c1(2)k+c2(9)k+c3(−18)k\rho(k)=c\_1 (2)^k+c\_2 (9)^k+c\_3 (-18)^kρ(k)=c1​(2)k+c2​(9)k+c3​(−18)k